

INTRODUCTION TO THEORETICAL METEOROLOGY



INSTITUT  
FÜR METEOROLOGIE U. KLIMATOLOGIE  
DER TECHN. HOCHSCHULE

HANNOVER · HERRENHAUSER STR. 2

Introduction to *Inv. S. 20/548*

# THEORETICAL METEOROLOGY

---

*SEYMOUR L. HESS*

*Florida State University*

INSTITUT F. MET. U. KLIMAT.  
TECHN. HOCHSCHULE HANNOVER

*B 6*

*A Holt-Dryden Book*

*HENRY HOLT AND COMPANY, NEW YORK*

**CONSTABLE & CO LTD**

10-12 ORANGE STREET, LONDON, W.C.2

# CONTENTS

	PAGE
PREFACE	vii
1. Introduction	1
1. <i>The Physical Foundation</i> , 1	
2. <i>The Goal</i> , 5	
3. <i>Units and Dimensions</i> , 5	
4. <i>The Earth</i> , 7	
5. <i>The Atmosphere</i> , 9	
PROBLEMS, 10	
2. The Equation of State	12
1. <i>The Variables of State</i> , 12	
2. <i>Charles' Law</i> , 13	
3. <i>Boyle's Law</i> , 15	
4. <i>The Equation of State of an Ideal Gas</i> , 15	
5. <i>Mixtures of Gases</i> , 18	
PROBLEMS, 19	
3. The Principles of Thermodynamics	20
1. <i>Work</i> , 20	
2. <i>Heat</i> , 22	
3. <i>The Law of Conservation of Energy</i> , 24	
4. <i>Internal Energy and Specific Heat Capacities of an Ideal Gas</i> , 27	
5. <i>Adiabatic Processes</i> , 30	
6. <i>Entropy and the Second Law of Thermodynamics</i> , 32	
7. <i>Summary of Thermodynamic Variables</i> , 36	
PROBLEMS, 37	

4. The Thermodynamics of Water Vapor and Moist Air 39
1. *Isotherms on an  $a, e$  Diagram*, 39
  2. *Thermal Properties of Water Substance*, 41
  3. *The Equation of State of Moist Air*, 43
  4. *Changes of Phase and Latent Heats*, 44
  5. *The Clausius-Clapeyron Equation*, 46
  6. *Adiabatic Processes of Saturated Air*, 51
  7. *Moisture Variables*, 58
- PROBLEMS, 64
5. Thermodynamic Diagrams 65
1. *General Considerations*, 65
  2. *The Emagram*, 67
  3. *The Tephigram*, 69
  4. *The Skew T-Log p Diagram*, 70
  5. *The Stüve Diagram*, 72
  6. *Choice of a Diagram*, 74
- PROBLEMS, 74
6. Hydrostatic Equilibrium 75
1. *The Hydrostatic Equation*, 75
  2. *Height Computations for Upper-air Soundings*, 77
  3. *The Hydrostatics of Special Atmospheres*, 80
  4. *Altimetry*, 86
  5. *Reduction of Pressure to Sea Level*, 88
- PROBLEMS, 90
7. Hydrostatic Stability and Convection 92
1. *General Considerations*, 92
  2. *The Dry and Moist Adiabatic Lapse Rates*, 92
  3. *The Parcel Method*, 95
  4. *Changes of Stability During Displacement of Layers*, 100
  5. *The Slice Method*, 103
  6. *Entrainment into Cumulus Clouds*, 106
  7. *The Bubble Theory*, 110
- PROBLEMS, 113

## 8. The Fundamental Physics of Radiation 114

1. *Nature of Radiation*, 114
  2. *Atomic and Molecular Spectra*, 115
  3. *Scattering*, 119
  4. *Black-Body Radiation*, 121
  5. *Radiative Transfer*, 125
- PROBLEMS, 127

## 9. Solar and Terrestrial Radiation 128

1. *The Nature of Solar Radiation*, 128
  2. *Geographical and Seasonal Distribution of Solar Radiation*, 131
  3. *Terrestrial Radiation*, 134
- PROBLEMS, 138

## 10. Applications to Radiation in the Earth-Atmosphere System 139

1. *The Basis of Elsasser's Method*, 139
  2. *The Elsasser Diagram*, 142
  3. *Radiative Heating and Cooling of Clouds*, 147
  4. *Infrared Radiative Cooling of the Atmosphere*, 148
  5. *Transformation of Maritime Polar Air into Continental Polar Air*, 149
  6. *Radiative Equilibrium and the Stratosphere*, 152
  7. *The Mean Annual Heat Balance*, 155
- PROBLEM, 160

## 11. The Equations of Motion on a Rotating Earth 161

1. *Inertial versus Noninertial Coordinate Systems*, 161
  2. *The Dynamical Equations in a Rotating Coordinate System*, 162
  3. *Gravitation versus Gravity*, 166
  4. *The Pressure-Gradient Force*, 169
  5. *Inertia Motion*, 170
  6. *Individual versus Local and Convective Derivatives*, 172
- PROBLEMS, 174

12. Horizontal Motion under Balanced Forces	175
1. <i>Equilibrium Motion</i> , 175	
2. <i>Geostrophic Flow</i> , 175	
3. <i>The Effect of Friction</i> , 179	
4. <i>Gradient Flow</i> , 180	
5. <i>Comparison of Geostrophic and Gradient Wind Values</i> , 185	
6. <i>Cyclostrophic Flow</i> , 187	
7. <i>Representation of the Pressure Gradient on Other Than Horizontal Surfaces</i> , 187	
8. <i>The Thermal Wind Equations</i> , 189	
PROBLEMS, 196	
13. Kinematics of Fluid Flow	198
1. <i>Kinematics versus Dynamics</i> , 198	
2. <i>Resolution of a Linear Velocity Field</i> , 198	
3. <i>Streamlines, Trajectories, and Streak Lines</i> , 201	
4. <i>The Stream Function</i> , 205	
5. <i>Circulation and its Relationship to Vorticity</i> , 208	
6. <i>The Equation of Continuity</i> , 212	
7. <i>The Complete Set of Equations Governing the Atmosphere</i> , 215	
PROBLEMS, 217	
14. The Mechanism and Influence of Pressure Changes	219
1. <i>The Tendency Equation</i> , 219	
2. <i>The Bjerknes-Holmboe Theory</i> , 221	
3. <i>The Isallobaric Wind</i> , 225	
PROBLEMS, 227	
15. Surfaces of Discontinuity	229
1. <i>Discontinuities</i> , 229	
2. <i>Fronts</i> , 230	
3. <i>Fronts in a Geostrophic Wind Field</i> , 232	
4. <i>Fronts as Zones of Transition</i> , 233	
5. <i>The Tropopause</i> , 234	
PROBLEM, 236	



16. Circulation, Vorticity, and Divergence Theorems	238
1. <i>The Circulation Theorem</i> , 238	
2. <i>Physical Interpretation of the Circulation Theorem</i> , 241	
3. <i>Selected Applications of the Circulation Theorem</i> , 244	
4. <i>A Vorticity Theorem</i> , 247	
5. <i>The Theory of Long Waves in the Westerlies</i> , 253	
6. <i>A Divergence Theorem</i> , 256	
PROBLEMS, 258	
17. The Fundamental Equations using Pressure as an Independent Coordinate	259
1. <i>Substitution of Pressure for Height</i> , 259	
2. <i>Horizontal Derivatives and Time Derivatives</i> , 260	
3. <i>The Equations of Motion</i> , 261	
4. <i>The Equation of Continuity</i> , 261	
5. <i>The Vorticity and Divergence Equations</i> , 262	
6. <i>Geostrophic and Thermal-Wind Approximations</i> , 263	
18. Viscosity and Turbulence	265
1. <i>The Fundamental Law of Viscosity</i> , 265	
2. <i>The Equations of Motion Including Viscosity</i> , 267	
3. <i>The Equations of Mean Motion in Turbulent Flow</i> , 269	
4. <i>Modeling and Dynamic Similitude</i> , 271	
5. <i>The Mixing-Length Theory</i> , 274	
6. <i>Vertical Structure of the Wind in the Lowest Turbulent Layer</i> , 276	
7. <i>Vertical Structure of the Wind above the Lowest Turbulent Layer</i> , 279	
8. <i>Diffusion of Other Properties</i> , 283	
PROBLEMS, 291	
19. Energy and Stability Relationships	292
1. <i>The Energy Equation</i> , 292	
2. <i>Internal and Potential Energies</i> , 294	
3. <i>Frictional Dissipation of Kinetic Energy</i> , 295	
4. <i>The Conversion of Potential and Internal Energies to Kinetic Energy</i> , 297	

5. <i>The Mechanical Generation of Kinetic Energy</i> , 302	
6. <i>Inertial Stability</i> , 306	
PROBLEMS, 310	
20. Numerical Weather Prediction	311
1. <i>Introduction</i> , 311	
2. <i>The Reasons for Richardson's Failure</i> , 314	
3. <i>The Basis of Modern Numerical Weather Prediction</i> , 316	
4. <i>Numerical Solution of the Law of Conservation of Vorticity</i> , 317	
5. <i>Integration by the Method of Relaxation</i> , 318	
6. <i>Establishment of the Future Boundary Values</i> , 320	
7. <i>Synopsis of the Procedure</i> , 321	
8. <i>Conclusion</i> , 322	
PROBLEM, 324	
21. The General Circulation	325
1. <i>Scale of Atmospheric Motions</i> , 325	
2. <i>Longitudinally Averaged Flow</i> , 326	
3. <i>Longitudinally Varying Flow</i> , 328	
4. <i>Constraints on Theories of the General Circulation</i> , 330	
5. <i>A Meridional Circulation Model</i> , 333	
6. <i>An Experimental Approach</i> , 336	
7. <i>The Angular-Momentum Balance</i> , 338	
8. <i>A Numerical Experiment</i> , 345	
Appendix I. Numerical Constants and Conversions	353
Appendix II. Derivation of Gauss's Divergence Theorem	354
Index	357
List of Symbols	End Papers